

Factoring Quadratic Expressions - SPECIAL CASES

Perfect Square Trinomials

This method is used when the first and last terms of a trinomial are perfect squares, and the middle term is twice the product of the two roots.

For example, given:

$$4x^2 + 12x + 9$$

$$(\quad)^2 = 4x^2$$

- the first term, $4x^2$, and the last term, 9 , are both perfect squares
- the middle term is twice the product of the their square roots, $2 \times (2x)(3)$

Perfect Squares	Example: $4x^2 + 12x + 9$
Write one bracket squared.	$(\quad)^2$
Write the square root of the first term at the beginning of the bracket.	$(2x \quad)^2$
Write the square root of the last term at the end of the bracket.	$(2x \quad 3)^2$
Put the sign of the middle term between the w terms in the brackets.	$(2x + 3)^2$

In general: $a^2x^2 + 2abx + b^2 = (ax + b)(ax + b) = (ax + b)^2$
 $a^2x^2 - 2abx + b^2 = (ax - b)(ax - b) = (ax - b)^2$

NOTE: Perfect square trinomials can also be factored using the simple or complex trinomial method. The two resulting brackets will be identical. Always write the final answer with a single bracket squared!

Examples: Factor fully.

a) $25x^2 + 20x + 4$

$$\begin{aligned} & \rightarrow (5x + 2)^2 \\ & (5x + 2)(5x + 2) \end{aligned}$$

b) $16x^2 - 24x + 9$

$$(4x - 3)^2$$

c) $4a^2 - 20a + 25$

$$(2a - 5)^2$$

d) $9m^2 - 42mn + 49n^2$

$$(3m - 7n)^2$$

Difference of Squares

This method is used when there are only *two terms separated by a minus sign*. Both terms must be *perfect squares* (i.e. have a square root which is a whole number).

Consider the expansion of:

$$(x+3)(x-3) =$$

$$(x-4)(x+4) =$$

Therefore, if $(x+y)(x-y) = x^2 - y^2$ then $x^2 - y^2 =$ _____

- i) A difference of squares expression always has exactly 2 numbers
 ii) Each of the terms is a perfect square
 iii) The terms are separated by a minus sign

Difference of Squares	Example: $4x^2 - 9$
Write two brackets.	$(\quad)(\quad)$
Write the square root of the first term at the beginning of each bracket.	$(2x)(2x)$
Write the square root of the second term at the end of each bracket.	$(2x \quad 3)(2x \quad 3)$
Separate the terms in one bracket with a + sign and in the other bracket with a - sign.	$(2x + 3)(2x - 3)$ ✓

Examples: (Remember to look for a *common factor* first!)

a) $x^2 - 25$

$$(x+5)(x-5)$$

b) $4x^2 - 25$

$$(2x+5)(2x-5)$$

c) $9x^2 - 49y^2$

$$(3x-7y)(3x+7y)$$

d) $2x^2 - 50y^2$

$$2(x^2 - 25y^2)$$

$$2(x-5y)(x+5y)$$

e) $x^6 - y^8$

$$(x^3)^2 = x^6$$

$$(y^4)^2 = y^8$$

$$(x^3 - y^4)(x^3 + y^4)$$

f) $3x^2 - 75$

$$3(x^2 - 25)$$

$$3(x+5)(x-5)$$